

1. A particle A has constant velocity $(3\mathbf{i} + \mathbf{j}) \text{ m s}^{-1}$ and a particle B has constant velocity $(\mathbf{i} - \mathbf{k}) \text{ m s}^{-1}$. At time $t = 0$ seconds, the position vectors of the particles A and B with respect to a fixed origin O are $(-6\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}) \text{ m}$ and $(-2\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) \text{ m}$ respectively.

(a) Show that, in the subsequent motion, the minimum distance between A and B is $4\sqrt{2} \text{ m}$. (6)

(b) Find the position vector of A at the instant when the distance between A and B is a minimum. (2)

(6)

(2)



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2. A car of mass 1000 kg is moving along a straight horizontal road. The engine of the car is working at a constant rate of 25 kW. When the speed of the car is v m s $^{-1}$, the resistance to motion has magnitude $10v$ newtons.

(a) Show that, at the instant when $v = 20$, the acceleration of the car is 1.05 m s $^{-2}$. (3)

(b) Find the distance travelled by the car as it accelerates from a speed of 10 m s $^{-1}$ to a speed of 20 m s $^{-1}$. (8)



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3. A small ball is moving on a smooth horizontal plane when it collides obliquely with a smooth plane vertical wall. The coefficient of restitution between the ball and the wall is $\frac{1}{3}$. The speed of the ball immediately after the collision is half the speed of the ball immediately before the collision.

Find the angle through which the path of the ball is deflected by the collision.

(8)



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4. At noon two ships A and B are 20 km apart with A on a bearing of 230° from B . Ship B is moving at 6 km h^{-1} on a bearing of 015° . The maximum speed of A is 12 km h^{-1} . Ship A sets a course to intercept B as soon as possible.

(a) Find the course set by A , giving your answer as a bearing to the nearest degree. (4)

(b) Find the time at which A intercepts B . (4)



Question 4 continued

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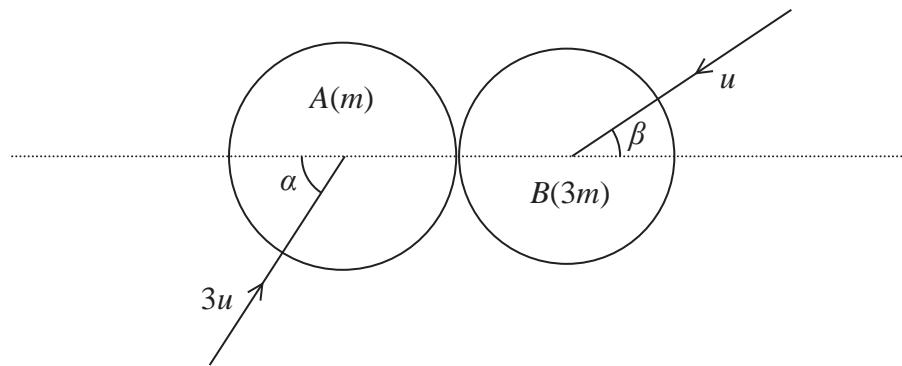


Figure 1

Two smooth uniform spheres A and B have equal radii. The mass of A is m and the mass of B is $3m$. The spheres are moving on a smooth horizontal plane when they collide obliquely. Immediately before the collision, A is moving with speed $3u$ at angle α to the line of centres and B is moving with speed u at angle β to the line of centres, as shown in Figure 1. The coefficient of restitution between the two spheres is $\frac{1}{5}$. It is given that $\cos \alpha = \frac{1}{3}$ and $\cos \beta = \frac{2}{3}$ and that α and β are both acute angles.

- (a) Find the magnitude of the impulse on A due to the collision in terms of m and u . (8)

(b) Express the kinetic energy lost by A in the collision as a fraction of its initial kinetic energy. (4)



Question 5 continued

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P 4 3 1 5 9 A 0 1 3 2 4

6. A particle of mass m kg is attached to one end of a light elastic string of natural length a metres and modulus of elasticity $5ma$ newtons. The other end of the string is attached to a fixed point O on a smooth horizontal plane. The particle is held at rest on the plane with the string stretched to a length $2a$ metres and then released at time $t = 0$. During the subsequent motion, when the particle is moving with speed v m s $^{-1}$, the particle experiences a resistance of magnitude $4mv$ newtons. At time t seconds after the particle is released, the length of the string is $(a + x)$ metres, where $0 \leq x \leq a$.

(a) Show that, from $t = 0$ until the string becomes slack,

$$\frac{d^2x}{dt^2} + 4 \frac{dx}{dt} + 5x = 0 \quad (3)$$

(b) Hence express x in terms of a and t .

(c) Find the speed of the particle at the instant when the string first becomes slack, giving your answer in the form ka , where k is a constant to be found correct to 2 significant figures.

(4)



Question 6 continued

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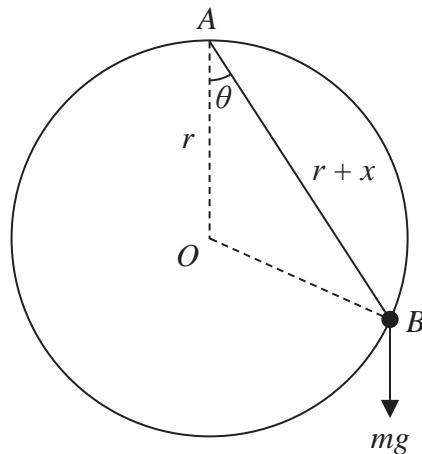


Figure 2

A bead B of mass m is threaded on a smooth circular wire of radius r , which is fixed in a vertical plane. The centre of the circle is O , and the highest point of the circle is A . A light elastic string of natural length r and modulus of elasticity kmg has one end attached to the bead and the other end attached to A . The angle between the string and the downward vertical is θ , and the extension in the string is x , as shown in Figure 2.

Given that the string is taut,

- (a) show that the potential energy of the system is

$$2mgr\{(k-1)\cos^2\theta - k\cos\theta\} + \text{constant} \quad (6)$$

Given also that $k = 3$,

- (b) find the positions of equilibrium and determine their stability.

(9)



Question 7 continued

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Q7

(Total 15 marks)

TOTAL FOR PAPER: 75 MARKS

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